

Prediction of optimum heat recovery using a calculation model based on non-linear, partial, relational and singular regressions to significantly reduce the amount of base data

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Abstract. Since 2020, non-linear multiple regressions have been acknowledged as a viable method for estimating optimal heat recovery.

With the usage of four additional influence parameters, a multiple regression would require an exponential number of around 8,000 data sets, whereas the partial and relational model requires only seven individual, singular regressions in combination (the original three plus additional four factors).

The non-linear multiple regressions were already discussed and partially considered as a parallel track in the revision process of the Eco Design Regulation EU 1253/2014. Due to the rapid fluctuation of energy prices as well as the need to accommodate other influencing factors, this study proposes an alternative modeling technique that utilizes a combination of non-linear singular and individual regressions. Change factors were used to represent the optimal temperature transfer efficiency and the required air velocity.

Furthermore, this study suggests calculating the pressure drop of the heat recovery without employing regressions but instead utilizing a known average pressure drop and calculating the optimal pressure drop based on alterations in the causal relationships.

A partial error analysis was used to evaluate the quality of the individual regressions based on the quality of the individual regressions, comparing the new partial and relational regression models with the non-linear multiple regression.

Keywords: optimal energy recovery, ventilation systems, energy efficiency, Life cycle assessment, energy calculation model

1 INTRODUCTION

The non-linear multiple regression analyses were developed to generate models for easily and universally predicting the optimal heat recovery (HR) of ventilation systems under various general conditions without the need for further individual optimization calculations.^{i ii iii}

Such a general approach was new and was not available before 2020.^{iv v}

The multiple regressions considered the following framework parameters:

- The outdoor air temperature in the winter, which corresponds to the geographic location,
- The extract air temperature represents the respective application of heat recovery,
- the operating time of HR,
- The balance boundary (HR used alone and other factors that affect heat and cold generation),
- The HR's load cases (full and part load).

To maintain a reasonable model complexity, the energy prices (namely the average price for electrical energy at 0.091 €/kWh and the price for heating at 0.043 €/kWh) were assumed to be constant. This assumption was made as these parameters only experienced minor changes during the review period (2008–2017) and could be assumed to be constant in the initial approach. A CO₂ price was included in the model to account for the impact of CO₂ reduction.

The outcomes of the multidimensional optimization under the specified general conditions, derived from two distinct load scenarios, served as the basis for establishing the regression model. Both a full load case, which corresponds to the design airflow, and a partial load case, which exhibits reduced airflow (70% of the nominal airflow), were considered.

Moreover, the optimizations were executed for two balance boundaries. On the one hand, considering heat recovery without further influences and on the other hand, with consideration of the reduction in heating- and cooling-generation systems. An air handling unit with heat recovery and an average airflow of 14,400 m³/h was considered to simulate operation, compared to an air handling unit without heat recovery.^{vi}

Three runtime models were used (2,350 h/a, 5,000 h/a and 8,760 h/a). A further data set of 8,766 h/a was included as a reference value for determining the regression, which is, however, identical to the values for 8,760 h/a.

The geographic location of heat recovery was represented by the outdoor air temperature (ODA). Using multidimensional optimization, the data sets were created with extract air temperatures of 18°C, 20°C, 22°C, and 24°C. This resulted in a total of 16 data sets for each scenario. All the data sets and the results of the multiple regressions are available at:

<https://data.mendeley.com/datasets/w7sw8njrz5/1> (DOI: 10.17632/w7sw8njrz5.1)

The independent variables (x_i), which describe the dependent variables (y_i) through regression analysis, are the outdoor air temperature, extract air temperature, and operating time of the heat recovery. The optimal temperature transfer efficiency, pressure drop, and air velocity were determined using multiple regressions.

The geographic location-specific definition of the requirements for HR has been discussed at the EU level since May 2020, serving as a "parallel track" to the revision of Regulation EU 1253/2014. The author's multiple regression, considering the CO₂ price

from Table 1, was explicitly suggested as an alternative method for revising the regulation.^{vii viii ix}

2 THE EXISTING NON-LINEAR MULTIPLE REGRESSION MODEL

Based on the previous analysis, it was observed that there exists a non-linear correlation between the outdoor air temperature and the temperature transfer efficiency of heat recovery. The air velocity and the pressure drop also exhibit a non-linear relationship with the outdoor air temperature. Therefore, linear regression was inapplicable in these instances as well. Furthermore, a simple regression was not possible because the dependent variables were determined by several independent variables. Additionally, the outdoor air temperature, the extract air temperature, and the operating time of the system are of crucial importance. Given the intricate interrelationships among variables, straightforward mathematical representations are unattainable, rendering a concise analytical resolution impracticable. IBM's SPSS statistical software was used to calculate multiple non-linear regressions.

The European Emissions Trading System (EU-ETS), which has served as the primary climate protection mechanism of the EU since 2005, has served as a benchmark for the CO₂ price, and it is imperative that it is also included. The CO₂ price at the time of review was 25 €/t (average for 2019). The CO₂ reduction was calculated from the reduction in emissions due to heat savings based on gas emission factors minus the additional emission costs for electrical energy based on the European mix for electricity and the emissions that occur during the production of the heat recovery system (gray CO₂ emissions).

The balance boundary encompasses not only heat recovery but also heating and cooling generation. This results in a subsequent correlation based on the lowest outdoor air temperature during winter, the extract air temperature, and the operating time of the heat recovery, as depicted in the following table.

Table 1: Multiple regression for the full load case and with an extended balance boundary with influence of the CO₂ price (25 €/t.)

Φ parameter		Δp parameter		w parameter	
ODA1	-1.023045	ODA1	-3.8557	ODA1	-0.003943
ODA2	-0.0581	ODA2	-0.2269	ODA2	-0.0006294
ODA3	-0.001338	ODA3	-0.008067	ODA3	-3.4451E-05
EXT1	3.9363	EXT1	-0.8103	EXT1	0.1129
EXT2	-0.0712	EXT2	0.1125	EXT2	-0.002670
OT1	0.004494	OT1	0.008117	OT1	-5.3070E-05
OT2	-2.3453E-07	OT2	-2.3739E-07	OT2	3.1804E-09

Temperature efficiency $aR^2 = 0.935$; pressure loss $aR^2 = 0.879$

Consequently, the program determines a third-degree polynomial that calculates the optimal temperature transfer efficiency based on the minimum outdoor air temperature during the winter. Furthermore, the extract air temperature and the operating time are considered by utilizing two additional 2nd-degree polynomial terms. Similar to that, polynomials can be used to calculate the pressure loss of the heat recovery at the optimum and the necessary air velocity. This leads to the following equation for the temperature transfer efficiency of the HR. The other influences are calculated analogously:

$$\Phi_{\text{opt}} = -1.023045 \cdot \text{ODA} - 0.0581 \cdot \text{ODA}^2 - 0.001338 \cdot \text{ODA}^3 + 3.9363 \cdot \text{EXT} \\ - 0.0712 \cdot \text{EXT}^2 + 0.004494 \cdot \text{OT} - 2.3453\text{E-}07 \cdot \text{OT}^2$$

2.1 Influence of volatile energy prices

For the observation period from 2008 to 2017, multiple regression models were developed based on statistical average values of energy prices. Nonetheless, the energy prices are currently subject to significant volatility. Hence, the outcomes of the economic efficiency and optimization calculations exhibit significant variations in accordance with the current and actual prices. The price of heat experienced a significant increase during the years 2021 and 2022. Particularly noteworthy is the increase in the price of gas by 56.1% in 2021 in comparison to the previous year. In 2022, the gas price initially rose significantly but then fell again significantly at the end of 2023.^x

The natural gas price was at the end of 2023 approximately at the same level used to calculate multiple regressions.^{xi} In 2024 the price rose, especially during the last month of 2024 significantly again.^{xii}

In contrast, electricity costs in Europe remained relatively stable at 8.6 cents per kWh in 2021 but soared to 16.0 cents per kWh in 2022. In 2023, there the price rose up to 19,4 cent per kWh, but fell in 2024 to 15,6 cents per kWh again.^{xiii} However, it is likely that the price of electricity will continue to follow in a manner like the price of gas, with a temporal delay.

It would be feasible to consider the possibility of incorporating additional multiple regressions to assess the impact of energy prices on the model. The database for multiple regression would be significantly expanded if energy prices were considered. With only five price pairs for heating and electrical energy, the database would increase from 16 to 400 data sets. Alternatively, it can be estimated how much each factor affects the overall result by using simple, singular non-linear regressions. However, this method may be less certain, but the relevant variables are neither correlated nor causally related in any way.^{xiv}

In the end, the price of CO₂ in particular experienced a significant rise in the year 2021. The average price for CO₂ certificates on the Leipzig Energy Exchange was around 60 €/t in 2021, compared to the 25 €/t they were traded in 2020. On January 7th, 2022, the price was 86.8 €/t.^{xv} and at the end of 2023 at 80.2 €/t. At the End of March 2025, the price is traded with around 69 €/t.^{xvi} Given the European Commission's emphasis on CO₂ reductions through the Green Deal, it is possible that variable CO₂ pricing could be a crucial factor in future planning.

3 PARTIAL REGRESSION MODEL

3.1 Basics influence factors for the partial nonlinear regression model

A model based on individual, singular regressions of the various influencing factors is proposed to reduce the amount of data and partially map the influences.

Individual influences for determining the optimal HR can be mapped using singular factors to generate a combined non-singular factor. The lowest outdoor air temperature in winter, for example, had a significant impact on the multidimensional optimizations (see Figure 1).^{xvii}

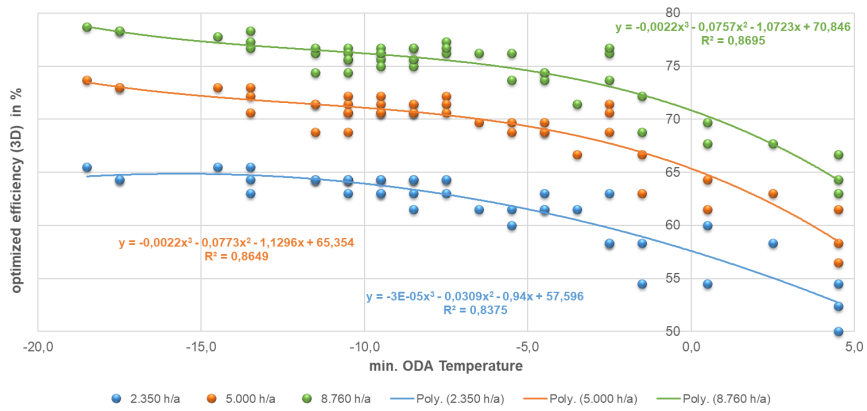


Fig. 1. Optimal temperature transfer efficiency considering investment savings and an extract air temperature of 20°C.

The design temperature of the outdoor air temperature in winter establishes a link to the geographic location of installation. It is evident that the outdoor air temperature corresponds to an optimal temperature transfer efficiency based on the operating time.

For the influences of the outdoor air temperature (ODA) in the range from -15°C to +2.5°C, the multidimensional optimizations resulted in the following cubic regression equations for the different operating times at the average location of Mannheim.

$$2,350 \text{ h/a with } \phi_{\text{opt}} = -0.00003 \cdot \text{ODA}^3 - 0.0309 \cdot \text{ODA}^2 - 0.940 \cdot \text{ODA} + 57.596$$

$$\text{with: } R^2 = 0.838$$

$$5,000 \text{ h/a with } \phi_{\text{opt}} = -0.0022 \cdot \text{ODA}^3 - 0.0773 \cdot \text{ODA}^2 - 1.1296 \cdot \text{ODA} + 65.354$$

$$\text{with: } R^2 = 0.865$$

$$8,760 \text{ h/a with } \phi_{\text{opt}} = -0.0022 \cdot \text{ODA}^3 - 0.0757 \cdot \text{ODA}^2 - 1.0723 \cdot \text{ODA} + 70.846$$

$$\text{with: } R^2 = 0.870$$

Furthermore, the following partial influences can be determined for the average location of Mannheim using additional multidimensional optimization:

a) Extract air temperature (EXT) in the range of 18°C to 28°C for:

$$2,350 \text{ h/a with } \phi_{\text{opt}} = 0.0007 \cdot \text{EXT}^3 - 0.1163 \cdot \text{EXT}^2 + 4.9843 \cdot \text{EXT} + 5.4472$$

$$\text{with: } R^2 = 0.993$$

$$5,000 \text{ h/a with } \phi_{\text{opt}} = 0.0005 \cdot \text{EXT}^3 - 0.0961 \cdot \text{EXT}^2 + 4.2782 \cdot \text{EXT} + 20.113$$

$$\text{with: } R^2 = 0.997$$

$$8,760 \text{ h/a with } \phi_{\text{opt}} = 0.0003 \cdot \text{EXT}^3 - 0.0764 \cdot \text{EXT}^2 + 3.5772 \cdot \text{EXT} + 32.744$$

$$\text{with: } R^2 = 0.998$$

b) Load (LOAD) (average airflow for demand-controlled systems) from 40% to 100% for:

$$2,350 \text{ h/a with } \phi_{\text{opt}} = -0.0007 \cdot \text{LOAD}^2 + 0.1390 \cdot \text{LOAD} + 57.630 \text{ with: } R^2 = 0.952$$

$$5,000 \text{ h/a with } \phi_{\text{opt}} = -0.0006 \cdot \text{LOAD}^2 + 0.1071 \cdot \text{LOAD} + 66.739 \text{ with: } R^2 = 0.935$$

$$8,760 \text{ h/a with } \phi_{\text{opt}} = -0.0002 \cdot \text{LOAD}^2 + 0.0375 \cdot \text{LOAD} + 74.675 \text{ with: } R^2 = 0.997$$

c) Supply air temperature (SUP) (depending on the internal load) from 16°C to 26°C for:

$$2,350 \text{ h/a with } \phi_{\text{opt}} = 0.0060 \cdot \text{SUP}^3 - 0.4227 \cdot \text{SUP}^2 + 9.8028 \cdot \text{SUP} - 9.5495$$

$$\text{with: } R^2 = 0.959$$

$$5,000 \text{ h/a with } \phi_{\text{opt}} = 0.0054 \cdot \text{SUP}^3 - 0.3742 \cdot \text{SUP}^2 + 8.5815 \cdot \text{SUP} + 7.3264$$

$$\text{with: } R^2 = 0.968$$

$$8,760 \text{ h/a with } \phi_{\text{opt}} = 0.0045 \cdot \text{SUP}^3 - 0.3145 \cdot \text{SUP}^2 + 7.250 \cdot \text{SUP} + 22.159$$

$$\text{with: } R^2 = 0.974$$

d) Heat price (HP) in the range of 0.04 €/kWh to 0.16 €/kWh for:

$$2,350 \text{ h/a with } \phi_{\text{opt}} = -57389 \cdot \text{HP}^4 + 30810 \cdot \text{HP}^3 - 6285.5 \cdot \text{HP}^2 + 654.04 \cdot \text{HP} + 46.164$$

$$\text{with: } R^2 = 0.998$$

$$5,000 \text{ h/a with } \phi_{\text{opt}} = -49463 \cdot \text{HP}^4 + 27964 \cdot \text{HP}^3 - 6039.4 \cdot \text{HP}^2 + 648.41 \cdot \text{HP} + 52.541$$

$$\text{with: } R^2 = 0.999$$

$$8,760 \text{ h/a with } \phi_{\text{opt}} = -63441 \cdot \text{HP}^4 + 34607 \cdot \text{HP}^3 - 7075.2 \cdot \text{HP}^2 + 694.89 \cdot \text{HP} + 56.894$$

$$\text{with: } R^2 = 0.998$$

e) Price for electrical energy (EP) in the range of 0.06 €/kWh to 0.36 €/kWh for:

$$2,350 \text{ h/a with } \phi_{\text{opt}} = -11.702 \cdot \text{EP} + 65.332 \text{ with: } R^2 = 0.851$$

$$5,000 \text{ h/a with } \phi_{\text{opt}} = -10.562 \cdot \text{EP} + 72.318 \text{ with: } R^2 = 0.866$$

$$8,760 \text{ h/a with } \phi_{\text{opt}} = -10.289 \cdot \text{EP} + 77.479 \text{ with: } R^2 = 0.930$$

The price of electrical energy and the load essentially determine the optimal air velocity necessary to achieve the optimum:

a) Price for electrical energy (EP) in the range of 0.06 €/kWh to 0.36 €/kWh for:

$$2,350 \text{ h/a with } w_{\text{opt}} = 14.324 \cdot \text{EP}^4 - 22.969 \cdot \text{EP}^3 + 13.713 \cdot \text{EP}^2 - 4.0998 \cdot \text{EP} + 1.3509 \\ \text{with: } R^2 = 0.987$$

$$5,000 \text{ h/a with } w_{\text{opt}} = 78.473 \cdot \text{EP}^4 - 83.359 \cdot \text{EP}^3 + 33.579 \cdot \text{EP}^2 - 6.6856 \cdot \text{EP} + 1.3766 \\ \text{with: } R^2 = 0.992$$

$$8,760 \text{ h/a with } w_{\text{opt}} = 56.772 \cdot \text{EP}^4 - 65.395 \cdot \text{EP}^3 + 28.073 \cdot \text{EP}^2 - 5.9155 \cdot \text{EP} + 1.3040 \\ \text{with: } R^2 = 0.987$$

b) Load (LOAD) (average airflow for demand-based systems) from 40% to 100% for:

$$2,350 \text{ h/a with } w_{\text{opt}} = 0.00005 \cdot \text{LOAD}^2 - 0.0140 \cdot \text{LOAD} + 1.9770 \text{ with: } R^2 = 0.997$$

$$5,000 \text{ h/a with } w_{\text{opt}} = 0.00006 \cdot \text{LOAD}^2 - 0.0166 \cdot \text{LOAD} + 2.0305 \text{ with: } R^2 = 0.996$$

$$8,760 \text{ h/a with } w_{\text{opt}} = 0.00005 \cdot \text{LOAD}^2 - 0.0138 \cdot \text{LOAD} + 1.8715 \text{ with: } R^2 = 0.998$$

The listed influencing factors show that this model accounts for significantly more extensive influences than the multiple regression model that was included in the draft revision of EU 1253/2014. In addition to the outdoor air temperature, the operating time, and the extract air temperature, the two energy prices for heating and electricity, the supply air temperature, and the load are also considered. This enables the mapping of not only the impact of fluctuating energy prices but also the partial load behavior and internal load through the maximum set point of the supply air temperature.

The inclusion of the supplementary CO₂ price factor was omitted as it is directly incorporated in the prices for heating and electricity, as both are defined similarly per kWh. Considering the reduction resulting from smaller heat and cold generation systems within the balance boundaries, it is permissible to disregard embodied emissions to produce the heat recovery system.

3.2 Combined influences for the partial nonlinear regression model

To reduce the quantity of data and partially map the influences, a proposed model is based on individual regressions of the various influencing factors. A factor is derived based on a change in the respective influencing parameter.

For instance, the regression analysis conducted on the extract air temperature and an operating time of 5,000 h/a yields the following results:

$$\phi_{\text{opt}} = 0.0005 \cdot \text{EXT}^3 - 0.0961 \cdot \text{EXT}^2 + 4.2782 \cdot \text{EXT} + 20.113$$

For the base value of 20°C, the optimal temperature transfer efficiency is $\phi_{\text{opt}} = 71.237\%$. If recalculated for 24°C, the optimum deviates to 74.348%. The effect of the elevated extract air temperature consequently increases by the factor $F_{\text{EXT } \phi} = 74.348 / 71.237 = 1.044$. So, by +4.4%.

With this particular model, the corresponding influence for each support value (ranging from 2,350 h/a to 5,000 h/a and 8,760 h/a) can be identified for each influence parameter based on the initial values ($F = 1.0$). The overall change in the optimal temperature transfer efficiency for each support value results from the multiplication of the individual factors.

$$F_{\text{total } \phi} = F_{\text{ODA } \phi} \cdot F_{\text{EXT } \phi} \cdot F_{\text{SUP } \phi} \cdot F_{\text{HP } \phi} \cdot F_{\text{EP } \phi} \cdot F_{\text{LOAD } \phi}$$

All factors are multiplied to generate the overall factor, which determines the efficiency of temperature transfer. In contrast to multiple regression, the polynomial components are not added but instead multiplied in the form of factors. Hence, each factor embodies the proportional impact of each individual parameter. With the usage of additional four influence parameters, a multiple regression would require an exponential number of around 8,000 data sets, whereas the partial and relational model requires only seven individual, singular regressions (the original three plus additional four factors) in combination.

The optimal temperature transfer efficiency derived from the multidimensional original optimizations is used as a basis; it is multiplied by the overall factor $F_{\text{total } \phi}$ to determine the desired optimum under the changed general conditions.

For each of the three operating times, there are factors $F_{\text{total } \phi}$, which are multiplied by the respective original optimum. A fourth base value of 8,766 h/a is used, but it is identical to the values for the operating time of 8,760 h/a.

A dynamic regression can be formed with these four reference values, allowing the time-dependent optimum to be calculated for any operating time (between 2,350 h/a and 8,760 h/a).

For example, from the following support values:

X (OT)	Y (ϕ)
2,350	67.960
5,000	74.433
8,760	78.942
8,766	78.942

Results in a dynamic cubic regression with $R^2 = 1.0$ for any operating time (OT) between 2,350 and 8,760 h/a:

$$\phi_{\text{opt}} = A \cdot \text{OT}^3 + B \cdot \text{OT}^2 + C \cdot L + D$$

With the dynamically generated polynomial parameters:

$$\begin{aligned} A &= -1.93969\text{E-}11 \\ B &= 1.18514\text{E-}07 \\ C &= 0.00239141 \\ D &= 61.93791204 \end{aligned}$$

The influence of air velocity can be determined, although only two influencing factors need to be considered.

$$F_{\text{total w}} = F_{\text{EP w}} \cdot F_{\text{LOAD w}}$$

A dynamic regression is employed to establish the optimal air velocity under a specified operating time, analogous to the efficiency of temperature transfer.

The model is shown in the attached Excel Tool:

<https://data.mendeley.com/datasets/f6f22nchnh/1> (DOI: 10.17632/f6f22nchnh.1).

4 CONSIDERATION OF THE UNCERTAINTIES OF THE REGRESSION MODEL

An error analysis can be used to determine the prediction accuracy of the temperature transfer efficiency by partially determining the influences of individual uncertainties.

For instance, the subsequent individual uncertainties arise in the form of the residuals (R^2) that result from the individual regressions:

	R	R ²	
F_{HP}	= 0.999	from 0.9982	for the price for heating
F_{EP}	= 0.939	from 0.8820	for the price of electricity
F_{ODA}	= 0.926	from 0.8573	for the outdoor air temperature
F_{EXT}	= 0.998	from 0.9957	for the extract air temperature
F_{SUP}	= 0.983	from 0.9669	for the supply air temperature
F_{LOAD}	= 0.998	from 0.9951	for the load

The partial derivative for the impact of the price of heating, for instance, is provided by:

$$\delta F / \delta F_{HP} \phi \cdot \Delta F_{HP} \phi = \Delta F_{HP} \phi \cdot F_{EP} \phi \cdot F_{ODA} \phi \cdot F_{EXT} \phi \cdot F_{SUP} \phi \cdot F_{LOAD} \phi$$

This usually results in probable uncertainties for the temperature transfer efficiency of an average value of around $R^2_{Total} = 0.809$ for the partial regression with a minimum of 0.792 and a maximum of 0.849, depending on the outdoor air temperature and operating time.

The partial regression error calculation for the air velocity yields a significantly lower level of uncertainty, with an approximate R^2 value of 0.988, which is a significant reduction from the previous method.

4.1 Comparison between multiple and partial non-linear regression

Since the chaining of the individual regressions is mathematically unusual, the comparison with multiple regression (Table 1) can provide information about the validity of the model based on singular partial regressions.

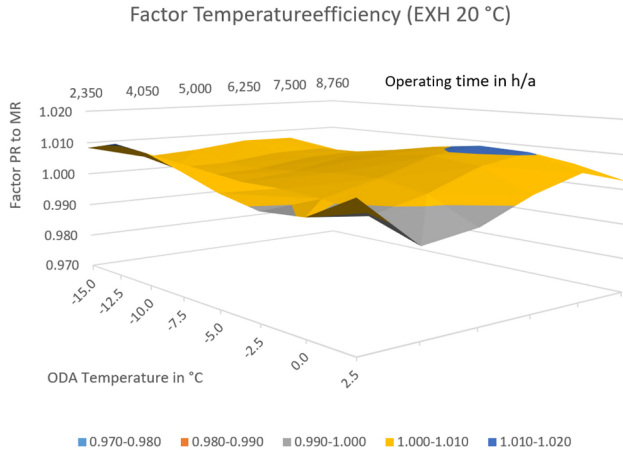


Fig. 2. Difference between the partial regression and the multiple regression for the temperature transfer efficiency in % at 20°C extract air temperature.

Figure 2 illustrates that the proportional deviation between the two regression models for an extract air temperature of 20°C is within the range of +1.1% to -0.7%.

At a high temperature of extract air (24°C), the relative deviation ranges from +0.8% to -2.0% (as depicted in Figure 3). At a low temperature of 18°C, the deviation increases to a maximum of +2.2% to 0.2% (as depicted in Figure 4).

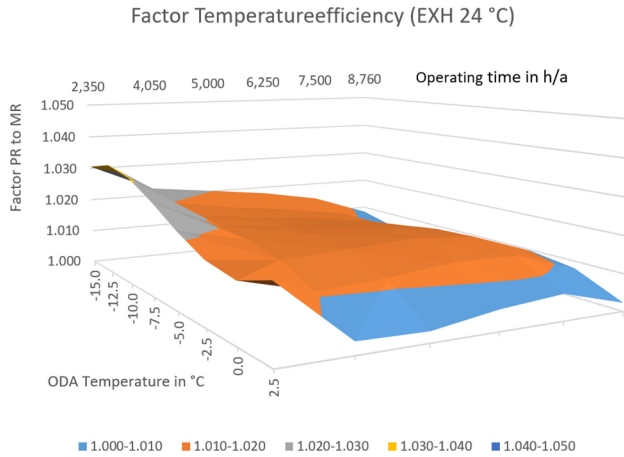


Fig. 3. Difference between the partial regression and the multiple regression for the temperature transfer efficiency in % at 24°C extract air temperature.

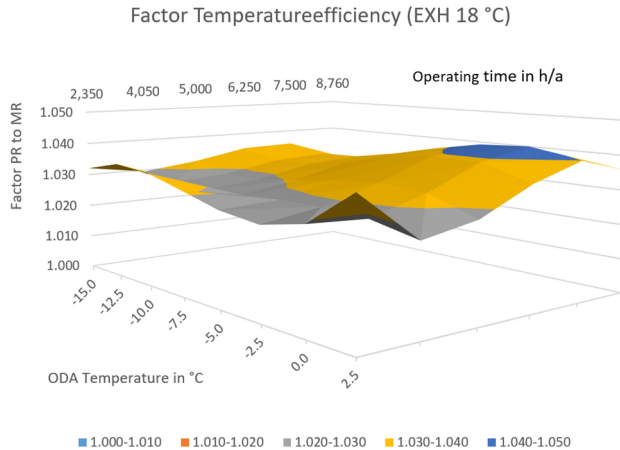


Fig. 4. Difference between the partial regression and the multiple regression for the temperature transfer efficiency in % at 18°C extract air temperature.

In general, the correlation between the outcomes of the two regressions is quite substantial. It should be noted that the carbon price was not considered in the partial regression model, but the temperature transfer efficiency is influenced by the CO₂ price, and it increases by approximately 2 to 3 %-points.^{xviii}

If the CO₂ price of 25 €/t, which was utilized to generate multiple regressions, is taken into account, the energy price for heating increases from 0.043 €/kWh to 0.05 €/kWh, and for electricity, from 0.091 €/kWh to 0.103 €/kWh. These adjustments arise from analyzing the emission factors for gas at 272 g CO₂ eq./kWh, which includes 35% distribution losses, and for electricity at 460 g CO₂ eq./kWh, in a manner similar to the multiple regression.

A comparison between the two models, incorporating the aforementioned adjusted energy prices, yields the deviations depicted in Figure 5.

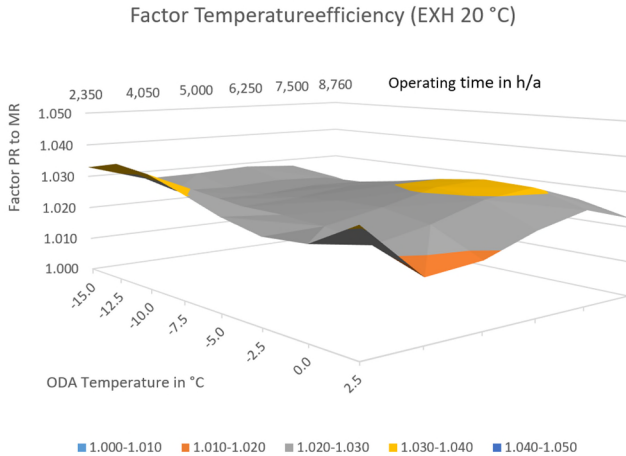


Fig. 5. Difference between the partial regression and the multiple regression for the temperature transfer efficiency in % at 20°C extract air temperature and energy prices adjusted for the CO₂ price.

At an extract air temperature of 20°C, the maximum deviations now lie in the range of +3.6% to +1.7%. At 24°C, the extract air temperature has a deviation between +3.3% and +0.2%, and at 18°C, the deviation is between +4.3% and +2.5%. This implies that partial regression yields values that are typically between 2 and 3 percentage points higher than multiple regression.

Nonetheless, this adjustment can solely be verified by examining the correlated variables equally implemented in both models, namely the variables outdoor air temperature, extract air temperature, and operating time.

The dynamically calculated regression quality of the partial regression approximates $R_{ges}^2 = 0.809$, whereas the ANOVA of the multiple regression yields $aR^2 = 0.935$. Consequently, it can be inferred that the combined partial regression model exhibits a signifi-

cantly lower prediction quality in theory compared to the multiple regression model. However, it must be acknowledged that the uncertainties of the other independent variables, such as supply air temperature, load, and energy prices, have already been incorporated into this model. If the supplementary influences were to be neglected, the R^2 would attain an average value of 0.896, with a range of 0.886 to 0.945.

Nonetheless, in predicting the optimal air velocity, the multiple regression ANOVA with a R^2 value of 0.725 yields a significantly less certain outcome as compared to the partial regression with aR^2 value of 0.988. The difference between the two regressions ranges from +1.8% to -4.9% at an extract air temperature of 20°C. The deviation ranges between +4.32% and -4.9% across the entire range of extract air temperature, spanning from 18°C to 24°C.

Given the optimal air velocity of approximately 1 m/s, it can be deemed that the absolute differences are very small.

The possibility of adjusting and validating the other independent variables, such as the supply air temperature and particularly the energy prices for heating and electricity, is not feasible. However, it can be assumed due to the high correlation between the model and the other influences, with R^2 values of 0.9982 for the price of heating, 0.882 for electricity, 0.9669 for supply air temperature, and 0.9951 for the load.

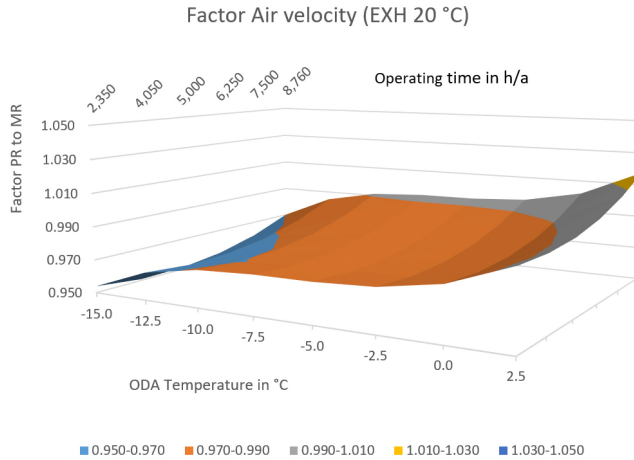


Fig. 6. Difference between the partial regression and the multiple regression for the air velocity in % at 20°C extract air temperature and energy prices adjusted for the CO₂ price.

The distribution of deviations between the two models suggests that there is no discernible systematic error; rather, it is assumed that there is a random distribution of errors.

To verify this thesis, the multiple regression model generated by investment changes in partial load operation was utilized as a further reference comparing both models, as this influence was also taken into account in both models.

Figure 7 demonstrates that the discrepancy between the two models ranges from -0.6% to +1.5% at an extract air temperature of 20°C. The entire range of extract air temperatures from 18°C to 24°C showed a maximum difference of -1.9% to +2.5% between the two models.

The difference between the two models ranged from a maximum of -7.3% to +5.3% when comparing the air velocity.

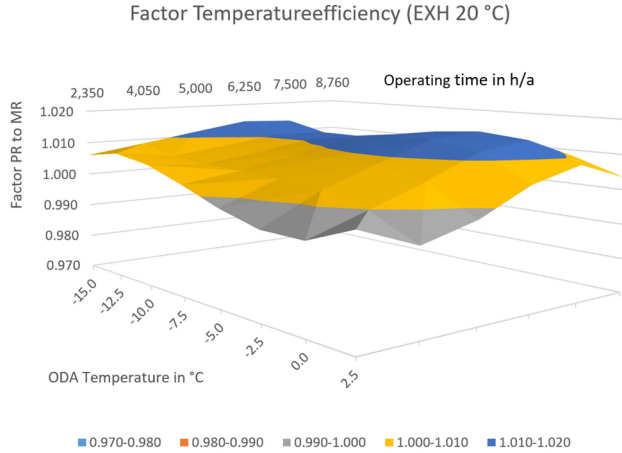


Fig. 7. Difference between the partial regression and the multiple regression for the air velocity in % at 20°C extract air temperature in partial load operation (70% of the nominal airflow).

4.2 Determination of the permissible pressure drop in the optimum

In addition to the temperature transfer efficiency and air velocity, the pressure loss of the heat recovery was calculated using multiple regression. Nonetheless, due to the impact of the pressure drop on the structural characteristics of the heat exchanger and the air velocity, it is suggested to derive the pressure drop primarily from the thermal efficiency and air velocity, rather than employing a regression approach.

Since all specifications refer to a balanced mass flow ratio, which is a realistic assumption,^{xix} the pressure drop can be determined depending on the dimensionless heat exchanger value NTU (number of transfer units):

$$NTU = \phi / (1 - \phi)$$

If the average temperature transfer efficiency is 73%,^{xx} this results in an NTU of:

$$NTU = 0.73 / (1 - 0.73) = 2.704$$

As a medium heat exchanger exhibits a pressure drop of $\Delta p_{\text{org}} = 176$ Pa, it is possible to determine the pressure drop in the optimal heat exchanger (Δp_{opt}) at constant air velocity as a function of NTU as follows:^{xxi}

$$\Delta p_{\text{opt}} = \Delta p_{\text{org}} \cdot \text{NTU}_{\text{opt}} / \text{NTU}_{\text{org}}$$

The pressure drop must be corrected as well, since the optimal air velocity fluctuates in relation to the current average velocity of $w_{\text{Lorg}} = 1.55$ m/s.^{xxii}

$$\Delta p_{\text{opt}} = \Delta p_{\text{org}} \cdot \text{NTU}_{\text{opt}} / \text{NTU}_{\text{org}} \cdot (w_{\text{Lopt}} / w_{\text{Lorg}})^{1,6}$$

Moreover, it is imperative to consider the alteration in k-value resulting from the alteration in air velocity, as:

$$\text{NTU} = k \cdot A / W$$

The k-value (heat transfer coefficient) is influenced by an air velocity that is usually lower at the optimum, which requires a larger heat-transferring area (A) to compensate for the reduction. The heat capacity flow ($W = m \cdot c_p$), however, remains unchanged. Additionally, this increases the pressure loss of the heat exchanger.

The k-value is adjusted according to Kaup^{xxiii} with:

$$k_{\text{opt}} = k_{\text{org}} \cdot (w_{\text{Lorg}} / w_{\text{Lopt}})^{0,4}$$

This implies that NTU must be corrected by this amount, resulting in the subsequent overall correlation:

$$\Delta p_{\text{opt}} = \Delta p_{\text{org}} \cdot \text{NTU}_{\text{opt}} / \text{NTU}_{\text{org}} \cdot (w_{\text{Lopt}} / w_{\text{Lorg}})^{1,6} \cdot (w_{\text{Lopt}} / w_{\text{Lorg}})^{-0,4}$$

Or summarized:

$$\Delta p_{\text{opt}} = \Delta p_{\text{org}} \cdot \text{NTU}_{\text{opt}} / \text{NTU}_{\text{org}} \cdot (w_{\text{Lopt}} / w_{\text{Lorg}})^{1,2}$$

The reduction in regression effort is significant for this calculation, as the pressure drop is directly related to the degree of temperature transfer efficiency and the air velocity.

4.3 consideration of the uncertainties of the pressure drop calculation

As the pressure drop is influenced by the two variables NTU and air velocity, the uncertainty of the pressure drop is calculated by analyzing the individual uncertainties of both variables, wherein NTU is directly influenced by the temperature transfer efficiency.

$$\Delta P = f(\text{NTU} \cdot w^{1,2}) = f(\phi / (1 - \phi) \cdot w^{1,2})$$

As the probable uncertainty for the temperature transfer efficiency is on average approximately $R^2 = 0.809$, and for the flow velocity at $R^2 = 0.988$ for the partial regression, the propagation of error for determining the pressure drop results in an average probable uncertainty of approximately $\pm 24\%$. For the reference conditions, an optimal temperature transfer efficiency of 71.5% has been calculated using a pressure drop of 96 Pa and an uncertainty of ± 23 Pa, at an air velocity of approximately 0.99 m/s, for Mannheim as the average location with an operating time of 5,000 h/a.

5 CONCLUSIONS

Based on the current findings, it is possible to establish a clear limitation to the HR specification. Notably, very short operating times render heat recovery uneconomical, especially when combined with small temperature differences (extract air temperature minus the geographic location-specific minimum outdoor air temperature).

The determinations made so far are based on these limits:

- Operating time ranging from 2,350 h/a to full-use operation with 8,760 h/a,
- Load from 40% to 100% of the rated airflow,
- Supply air temperature level of at least 14 °C,
- Extract air temperature level of at least 18 °C and a
- minimum outside air temperature, ranging from -15 °C to a maximum of +2.5 °C.

The maximum temperature of the extract air was restricted to 28°C; as exceeding this threshold, the set points for the supply air temperature (maximum 22°C) impose a limit on the heat recovery. If the case falls outside these limits, HR may still be economical in individual instances.

In such scenarios, however, a conclusive assessment can only be achieved through individual optimization. In this particular instance, the newly developed model, which is based on partial and relational regressions, permits the consideration of individual energy prices, which were standardized as average values across Europe in the previous model.

The new partial and relational regression considers the impact of the heating price, which spans from 0.04 €/kWh to 0.16 €/kWh, as well as the electricity price, which spans from 0.06 €/kWh to 0.36 €/kWh. This significantly broadens the scope of application for the model.

In actual individual instances, however, the values may exhibit significant variances, particularly in the event of persistent rises in CO₂ prices, which necessitate inclusion in the heat or electricity prices, thereby significantly altering the outcome of the individual optimization.

Outdoor air temperatures below $-15\text{ }^{\circ}\text{C}$ were deliberately not taken into account due to the rare occurrence of such temperatures, and the anti-icing protection also hinders heat transfer at these low temperatures.

However, there are also applications that make heat recovery uneconomical. As an example, data centers are mentioned here. In most cases, there is no use for the heat that could theoretically be recovered, despite the high temperatures. This is also the case in systems that feature free cooling, such as those found in exhibition halls. In some industries, process heat is available as waste heat from other processes at a higher or more economically sensible temperature level, taking precedence over heat recovery. Regrettably, in numerous instances, the unused heat is ineligible, rendering heat recovery a nonsensical endeavor.

It is important to emphasize that all regressions and enhancements rely on the assumption that heat can be utilized in the same process (or in another available process) and is required at the same time. It is essential to have a different perspective on HR, as it can't be assessed always the same way. The economic or ecological potential is prevented from being wasted with a heat recovery that is too small or too large by using regressions.

Ultimately, it is imperative to acknowledge with self-criticality that appropriate optimization is only feasible in individual instances, based on individual economic and/or ecological considerations, taking into account all the general conditions. Even then, the calculations are based on general conditions that are subject to change in the future.

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NOMENCLATURE

A	heat transfer area [m^2]
ANOVA	analysis of variance (one-factor analysis)
aR^2	regression quality of multiple regression
EU-ETS	European Emissions Trading System
EP	price for electricity [$\text{€}/\text{kWh}$]
EXT	extract air temperature [$^{\circ}\text{C}$]
EXT1-2	regression parameters for the extract air temperature, first to second order
F	factor [./.]
Δp	pressure drop [Pa]
HP	price for heating [$\text{€}/\text{kWh}$]
HR	heat recovery
k	heat transfer coefficient [$\text{W}/\text{m}^2/\text{K}$]
LOAD	load, average airflow [%]
NTU	number of transfer units [./.]

OT	operating time [h/a]
OT1-2	operating time regression parameters, first to second order
Φ	temperature efficiency or heat recovery [./.]
ODA	outdoor air temperature [°C]
ODA1-3	regression parameters for the outdoor air temperature, first to third order
R ²	regression quality [./.]
SUP	supply air temperature [°C]
W	heat capacity flow [W/K]
w	air velocity in the narrowest cross-section [m/s]
x	independent variable
y	dependent variable

INDEX DIRECTORY

EP	price for electricity
EXT	extract air
HP	price for heating
I	index
LOAD	load, average airflow
ODA	outdoor air
opt.	optimized
org.	original
SUP	supply air
total	total
w	air velocity
Φ	temperature efficiency or heat recovery

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